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great work, which may be said to have begun indirectly in 1528, and directly in 1744, was at last finished. The letter announcing this fact is in my collection and reads as follows:

> To General Sanson, Director of the Depôt of War:
>
> Count Cassini, Member of the Institute, of the General Council, and of the Electoral College of the Department of the Loire:
>
> *General*, I venture to flatter myself that, under the direction of a brave and loyal soldier like yourself, I can see, at the General Depôt of War, the end of the interminable affair of the Carte Générale de la France.
>
> You have before you or your representative the latest decree of the Council of State that governs the acts of those engaged upon the map of France. I have examined carefully the report which they have made, covering the two years under the Minister of War, and I am returning it with the recommendation that a general settlement be made in accordance therewith.
>
> This report strengthens still more the agreements between the minister and those who have acted for the Compagnie de la Carte Générale de la France, concerning the distribution of the maps to the subscribers and to the associates.
>
> The decree places the matter hereafter in your hands. It is for the Depôt of War to cancel the various obligations, and now it becomes the real owner of the material which has been prepared under the act relating thereto. . . .
>
> I cannot retire from the work without taking the opportunity of assuming the honor of offering to you the testimony of the distinguished consideration which you have inspired within me and which I shall retain as long as I live.
>
> General,
>
> Your very humble
> CASSINI
> *Member of the Institute.*
>
> 1st Messidor, an XI (June 19, 1803).

The letter bears the usual memoranda of reference for report, and of filing. The omitted portion has no special interest, referring only to details of distribution.

Thus ended a great scientific work that had extended over so many years,—the greatest work of its kind that had ever been undertaken up to that time, now more than a century ago.

QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

DISCUSSIONS.

The first discussion was suggested by Problem 2863 [*1920*, 482];[1] it appears in this department because the method of attack and the incidental results give it an independent interest. Mr. T. L. Bennett discusses certain properties of roulette curves by the method of circular coördinates. He has presented the material in such form as to imply no previous acquaintance with this method; and indeed his discussion gives an adequate introduction to the study of the properties of circular coördinates.[2]

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[1] See the problem department of this issue of the MONTHLY.

[2] The idea of circular coördinates seems to have originated with Plücker. Some articles dealing with the subject are: "Ueber Kreiscoordinaten" by W. Stammer, *Journal für die reine und angewandte Mathematik*, vol. 44, 1852, pp. 295–316; "On some applications of circular coordinates" by F. Franklin, *American Journal of Mathematics*, vol. 12, 1890, pp. 161–190; "Sundry metric theorems concerning* n *lines in a plane" by F. H. Loud, *Transactions of the American Mathematical Society*, vol. 1, 1900, pp. 323–338; and F. Morley's article, pp. 97–115, referred to below. —EDITOR.

Mr. R. S. Underwood proposes and answers, in the second discussion, an interesting question concerning trigonometric functions: when will the sine or cosine of an angle rationally expressible in degrees be rational? It may be remarked that it is well known that $\sin \theta$ and θ, when θ is expressed in radians, can never simultaneously be even algebraic, much less rational numbers, for $\theta \neq 0$, and that a similar statement holds for each of the other elementary trigonometric functions. Proofs of these theorems are somewhat intricate and are closely related to the famous proofs of the transcendentality of e and π. With the same notation, Mr. Underwood's theorem may be said to deal with the simultaneous rationality of θ and $\sin \pi\theta$. This belongs to a simpler order of ideas and may be settled by an easy and direct method of attack. It has perhaps a more obvious interest for the field of collegiate mathematics than the more difficult question mentioned above. Mr. Underwood leaves open the nature of the result for the tangent and cotangent; without doubt the facts can be ascertaine sid bymilar methods in these cases.

I. A Theorem on Hypocycloids, by the Method of Circular Coördinates.

By T. L. Bennett, University of Illinois.

Let the plane be referred to rectangular coördinates (X, Y). The circular coördinates (x, y) of the point (X, Y) are defined to be $x = X + iY$, $y = X - iY$.

A complex number $a + ib$, for which $a^2 + b^2 = 1$, is called a *turn*. A turn may be written $e^{i\theta}$, where $\theta = \cos^{-1} a = \sin^{-1} b$. It is easily shown that the conjugate of a turn is its reciprocal, and that any product of turns is a turn. All points which have turn coördinates lie on a unit circle about the origin. A turn will be denoted by some form of the letter t.

A curve may be represented by a map equation, $x = f(t)$. When t assumes turn values we get values of x, thus determining the points of the curve, because y is known as soon as x is known, being the conjugate of x. If $f(t)$ is a rational function, there is thus established a definite correspondence between the points of the unit circle and the points of the curve. If in the map equation t is replaced by any function of t which is a turn when t assumes turn values, the curve is not changed, as we have merely a different distribution of the parameter along the curve. With every map equation, $x = f(t)$, is associated another equation, namely, $\bar{x} = y = \overline{f(t)}$. If t be eliminated between these two equations, the result is the equation of the curve in circular coördinates.

The equation in circular coördinates of any straight line may be written in the form $t_1 x + y = r t_1$. The coefficient t_1 is called the *clinant* of the line, and determines the direction of the line thus: the angle which the line makes with the positive real axis is found to be

$$\phi = \frac{-1}{2i} \log (-t_1).$$

If a curve is defined by the map equation $x = f(t)$, it can be shown that the clinant of the tangent at any point is $-(dy/dt)/(dx/dt)$.